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Elementary Set Theory

Notation –

$\{\ \}$ enclose a set

$\{1, 2, 3\} = \{3, 2, 2, 1, 3\}$ because a set is not defined by order ~~of~~ or multiplicity.

$\{2, 4, ---\} = \{x \mid x$ is an even natural number$\}$ because two ways of writing a set are equivalent.

ϕ is the empty set.

$x \in A$ denotes x is an element of A.

$N = \{1, 2, 3, --\}$ are the natural numbers.

$Z = \{---, -2, -1, 0, 1, 2, ---\}$ are the integers.

$Q = \{\frac{p}{q} \mid p, q \in Z$ and $q \neq 0\}$ are the rational nos.

R are the real nos.

Axiom 1.1:– Axiom of elementary extensionality

Let A, B be sets. If $(\forall x) x \in A$ iff $x \in B$, then $A = B$.

Definition 1.1:– (Subset): Let A, B be sets. Then A is a subset of B, written $A \subseteq B$ iff $(\forall x)$ if $x \in A$ then $x \in B$.

Theorem 1.1:- If $A \subseteq B$ and $B \subseteq A$, then $A = B$

Proof:- Let x be arbitrary.

Because $A \subseteq B$ if $x \in A$ then $x \in B$

" $B \subseteq A$ if $x \in B$ then $x \in A$

Hence, $x \in A$ iff $x \in B$, thus $A = B$. ; proved

Def. 1.2 (Union):- Let A, B be sets. The union $A \cup B$ of A and B is defined by

$x \in A \cup B$ if $x \in A$ or $x \in B$.

Theorem 1.2:- $A \cup (B \cup C) = (A \cup B) \cup C$

Proof:- Let x be arbitrary.

$x \in A \cup (B \cup C)$ iff $x \in A$ or $x \in B \cup C$

iff $x \in A$ or ($x \in B$ or $x \in C$)

iff $x \in A$ or $x \in B$ or $x \in C$

iff ($x \in A$ or $x \in B$) or $x \in C$

iff $x \in (A \cup B)$ or $x \in C$

iff $x \in (A \cup B) \cup C$; proved.

Def. 1.3 (Intersection):- Let A, B be sets. The intersection $A \cap B$ of A and B is defined by $x \in A \cap B$ iff $x \in A$ and $x \in B$.

Theorem 1.3:- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Proof:- Let x be arbitrary. Then $x \in A \cap (B \cup C)$ iff $x \in A$ and $x \in B \cup C$

iff $x \in A$ and ($x \in B$ or $x \in C$)

iff ($x \in A$ and $x \in B$) or ($x \in A$ and $x \in C$)

iff $x \in A \cap B$ or $x \in A \cap C$

iff $x \in (A \cap B) \cup (A \cap C)$; proved